Population Curve **Date: ____________**

Subject: Science, math
Grade: 7-12
Time: 50 minutes

Topic: Using graph skills to investigate population growth
Designer: Nathan Kimball

Stage 1 – Desired Results

Lesson Overview: This activity exercises students graph reading and interpretation skills in the context of the science of population growth. Students will need to integrate the skills that are taught at a more elementary level in the other Smartgraphs and Graph Literacy activities. The activity introduces and contrasts three models of population growth: linear, exponential, and logistic and then focuses on the logistic model (also known as the sigmoidal or S-curve). Students learn about the concept of a carrying capacity and the three phases of logistic growth (lag, exponential, and stationary). They use line fitting to determine the exponential phase which helps them to segment the curve into three phases. A population experiment is performed using the aquatic plant duckweed. Students need to make predictions for this experiment, and the results are presented which are probably quite unexpected for students. They then think deeply about these outcomes and having a model of population can help design a better experiment and make better predictions.

Standards Addressed: The Common Core State Standards (CCSS) for mathematics do not directly address graph reading skills as content standards, but rather, embed them as pervasive and flexible skills of mathematical communication and representation in the Standards for Mathematical Practice. Below are excerpts from the Practices that this activity addresses.

CCSS.Math.Practice.MP4	**Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

CCSS.Math.Practice.MP7

Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

CCSS.Math.Practice.MP2

Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations… Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Students will be able to:	**Essential Questions**:
• Distinguish three growth models: linear, exponential, and logistic. • Use the technique of linear graph fitting to identify the breakpoints in the logistic curve and the phases of logistic growth. • Compare a real-world experiment outcome to a mathematical model. • Discuss the importance of having a mathematical model in making an informed prediction.	• Why do breakpoints in a population graph signal important events for a population? • What is carrying capacity for a population, and why is it important? • How does having the logistic model help making a prediction?

Stage 2 – Lesson Plan

Lesson preparation

- Students should understand linear graphs and equations, including expressing lines in point-slope form.
- Students should be familiar with exponential equations (In the Smartgraphs Algebra activities there are three activities under Exponential Functions that provide good preparation: http://concord.org/projects/smartgraphs#curriculum)
- The Smartgraphs activity African Lions compliments this activity in that it introduces the logistic curve and the concept of carrying capacity. You may want start with African Lions, and use this activity to deepen students understanding of the logistic curve and see how that theory could be used in the context designing an experiment that they could perform.

Procedure

1. Students will use computers/tablets for this activity (30-40 minutes).

2. The mathematics of populations is a great way to have students relate mathematics to the real world. Try leading students in a discussion of the questions below.

3. To assess students' comprehension of the logistic curves without using a detailed formula, have them do the graph sketching exercise below.

4. This activity provides an introduction to duckweed experiments. They are fun to do, and they help students understand populations in a new way. Many explanations for experiments are available on the web and elsewhere. If you have time, undertake a class duckweed experiment. Have students sketch logistic curves for predictions.

Further discussion questions

1. **Why do you think Malthus thought that the food supply grows linearly?**
   Have students think about how food is produced. Linear food growth assumes that food grows in proportion to the area of land farmed, for instance, that two acres of land will produce twice the food of one acre. In Malthus' time, bringing more land under cultivation was the best way to increase the food supply.
2. **Was Malthus right? The world's population continues to grow, and, in fact, almost all of the world's land that can be farmed is already farmed (without cutting rainforests, etc.), yet the world's food supply continues to grow. How is this possible?**
   Perhaps this is not a truly mathematical question, but it shows the power of having mathematical models. The linear model does not produce adequate food for the growing population, so other solutions are needed. Increases to the productivity of the land has allowed food supply to keep pace with population, by in large. (A large number of people are hungry, but this is an economic and distribution problem.) The real question is, How long can we continue to grow more food in the same space? Will Malthus be right eventually? The answer to this is not known, but only using mathematical models can we predict the need.
3. **Let say that you found that your duckweed grew exponentially (not logistically). If you had found that duckweed grew with a doubling rate of three days, and the beaker of duckweed covered one-quarter of the area of the duckweed, in how many days would the duckweed cover the whole beaker?**
   Students should be able to do this in their heads. With this question, you can find out if they have understood doubling rate. The answer is six days. After three days, half would be covered, and in the following three days, it would be completely covered.

Sketching Logistic Graphs

Name: _____________ Date: _________

Sketching graphs is a quick and easy way to convey mathematical ideas. For each question below, sketch the axes and the graphs that meet the criteria.

1. On one graph, sketch two logistic curves that have the same carrying capacity but different growth rates. Identify the graph with the greater growth rate.

2. On one graph, sketch two logistic curves that different carrying capacities but similar growth rates?